Multiobjective Decision Making in a Fuzzy Environment with Applications to Helicopter Design

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Much decision making in the real world takes place in an environment in which the goals, constraints, and consequences of possible actions are not known precisely. The tools of fuzzy set theory can be used to deal with such imprecision in a quantitative manner. This use and effectiveness of fuzzy theories in the formulation and solution of design problems are developed and described herein through application to two types of helicopter design problems involving multiple objectives. The first problem deals with the determination of optimum flight parameters to accomplish a specified mission in the presence of three competing objectives. The second problem addresses the optimum design of the main rotor of a helicopter involving eight objective functions. The tools of fuzzy set theory have been used to model the vague and imprecise information in the formulation of these problems. A method for solving the resulting fuzzy multiobjective problem using nonlinear programming techniques is presented. Results obtained using fuzzy formulation are compared with those obtained using crisp optimization techniques. The outlined procedure should be useful in engineering design situations in which uncertainty arises about the preciseness of permissible parameters, degree of credibility, and correctness of statements and judgments.

Nomenclature

f = vector of objective functions
f_j = jth objective function
IGE = in-ground effect
k = number of objective functions
OEI = one engine inoperative
OGE = out-of-ground effect
V_{NE} = indicated airspeed
X = set of feasible design variables
X_0 = starting design vector
X_i^* = optimum design vector for ith objective
Λ = fuzzy intersection
$\mu_{(\ )}$ = grade of membership of ( )

Superscripts

l, u = lower and upper bounds, respectively

Introduction

IN modeling real-world decision-making problems, a designer is often forced to state a problem in precise mathematical terms rather than in the frequently imprecise terms of the real world. The relationships and statements used for description may be imprecise not because of randomness but because of inherent fuzziness in the system. Fuzziness is a type of imprecision associated with fuzzy sets in which there is no sharp transition from membership to nonmembership. Further, with increasing system complexity, one's ability to make precise and significant statements concerning a given system diminishes.[1] Consequently, the closer one examines a real-world problem, the fuzzier its description becomes. Fuzzy set theories can effectively model such domains in which the description of activities and observations are "fuzzy," in the sense that there are no sharply defined boundaries of the set of activities or observations to which the descriptions apply. These theories enable one to structure and describe activities that differ from each other vaguely, to formulate them in models, and to use these models for problem solving and decision making.

Fuzzy set theory was initiated by Zadeh[2] in 1965. For some ten years thereafter, the mathematics of the subject were developed, but few applications resulted. During the last decade, these theories have been applied to various areas such as artificial intelligence, control, image processing, pattern recognition, robotics, and psychology. The first application of fuzzy theories to decision-making processes was presented by Bellman and Zadeh.[3] This paper prescribed basic concepts and definitions associated with a decision-making process in a fuzzy environment. Since then, these conceptual techniques have been employed to formulate and solve several mathematical programming problems.

Zimmerman has applied fuzzy optimization techniques to linear programming problems with single[4] and multiple objectives.[5] An application of these theories to preemptive and Archimedian versions of goal programming problems has been presented by Hannan.[6] Wang and Wang[7] have used the method of level cut solutions for the fuzzy optimum design of structures. Furuta et al.[8] have built an expert system based on fuzzy sets for the design and damage assessment of civil engineering structures. Rao has employed fuzzy optimization techniques for the design of mechanical[9] and structural systems.[10] An application of these techniques to multiobjective, multiple-attribute decision-making[11] problems has been presented by Yager.[12] The concept of efficient and weakly efficient solutions in the context of fuzzy multiobjective problems has been discussed by Feng[13] and Negotia.[14]

We develop and present methods for transforming imprecise linguistic statements associated with multiobjective rotorcraft design problems into equivalent crisp mathematical statements using fuzzy logic. For example, there may be several vaguely stated requirements during the preliminary stages

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of design of a helicopter, such as: "The new design should weigh *substantially* less than an existing design," and "The hover ceiling of the new design should be in the *vicinity* of 8000 ft." The italicized words are the sources of fuzziness in these statements. Fuzzy theories are used to translate imprecise and vague information present in the objective and constraint functions to formulate fuzzy objective and constraint functions. The fuzzy constraints define a fuzzy feasible domain in the design space. Each of the fuzzy objective functions defines its own optimum solution by a fuzzy set of points in a small fuzzy optimum subregion. An overall optimum solution that gives the highest degree of satisfaction with respect to all of the objectives and constraints is also identified.

Decision Making in an Ill-Structured Situation

Traditional optimization schemes assume that all of the design data are precisely known, that the constraints delimit a well-defined set of feasible decisions, and that the objectives are clearly defined and easy to formulate. An optimal decision is that combination of decision variables X^* that gives the "highest degree of satisfaction" for the objective function $f(X)$.

For a problem involving uncertainty and fuzziness in the input data, this notion of optimization needs to be modified. The objectives and constraints constitute a class of alternatives whose boundaries are not well defined. To deal with this imprecision quantitatively, the tools of fuzzy set theory can be used. The fuzzy objective functions and constraints are characterized by their membership functions. Since the overall optimization process requires simultaneous satisfaction of the objective function and the constraints, a decision or selection of a set of design variables is made by assuming that the constraints are noniterative and that the logical *and* operator corresponds to intersection. Because of the symmetry of the goals and constraints procedure, there is no longer any distinction between the objectives and the constraints of a decision process.

Consider a crisp nonlinear vector minimum problem of the form:

$$\text{Minimize } f(X)$$

subject to

$$g_j(X) \le b_j \qquad j = 1,2,\ldots,m \tag{1}$$

where

$$X = (x,x_2,\ldots,x_n)^T \tag{2}$$

$$f(X) = [f_1(X),f_2(X),\ldots,f_k(X)]^T \tag{3}$$

The fuzzy equivalent of the crisp nonlinear programming problem [Eq. (1)] can be stated as

Find X, which

$$\text{Minimizes } f(X)$$

subject to

$$g_j(X) \tilde{\in} \bar{G}_j, \qquad j = 1,2,\ldots,m \tag{4}$$

where $\bar{G}_j$ denotes the allowable interval for the constraint function g_j, $\bar{G}_j = [g_j^l, g_j^u]$. The bar over a symbol indicates that the expression or variable contains fuzzy information. The fuzzy constraint $g_j \tilde{\in} \bar{G}_j$ indicates that g_j is a member of $\bar{G}$ such that $\mu_{\bar{G}}(g_j) > 0$. A fuzzy feasible region is defined by considering all of the constraints as

$$\bar{R} = \bigcap_{j=1}^{m} \bar{G}_j \tag{5}$$

This gives the overall degree of satisfaction of design vector X with respect to all the constraints. A fuzzy decision is defined as the confluence of the goal ($\bar{F}$) and the constraints ($\bar{G}_1,\bar{G}_2,\ldots,\bar{G}_m$) as

$$\bar{D} = \bar{F} \cap \bar{G}_1 \cap \bar{G}_2 \cap \ldots \cap \bar{G}_m \tag{6}$$

and in terms of membership values as

$$\bar{D} = [\mu_f(X)] \wedge \{\bigcap_{j=1}^{m} \mu_{\bar{G}_j} [g_j(X)]\} \tag{7}$$

The optimum solution X^* is one for which the membership is maximum, i.e.,

$$\mu_{\bar{D}}(X^*) = \max \mu_{\bar{D}}(X), \ X \in \bar{D} \tag{8}$$

where

$$\mu_{\bar{D}}(X) = \min [\mu_f(X),\mu_{g_1}(X),\ldots,\mu_{g_m}(X)] \tag{9}$$

When the analogy from the single-objective function case is generalized, the resultant decision for a problem with k goals $(f_1,f_2,\ldots,f_k)$ and m constraints $(g_1,g_2,\ldots,g_m)$ is given as

$$\bar{D} = f_1 \wedge f_2 \wedge \cdots \wedge f_k \wedge g_1 \wedge g_2 \wedge \cdots \wedge g_m \tag{10}$$

In terms of corresponding membership values for the goals and the constraints, the resultant decision is

$$\bar{D} = \left[\bigcap_{i=1}^{k} \mu_{f_i}(X)\right] \wedge \left\{\bigcap_{j=1}^{m} \mu_{\bar{G}_j}[g_j(X)]\right\} \tag{11}$$

with

$$\mu_{\bar{D}}(X) = \min_{i,j} [\mu_{f_i}(X),\mu_{g_j}(X)] \tag{12}$$

Computational Procedure

The efficient solution of the fuzzy multiobjective problem given by Eq. (4) is determined by 1) finding the solutions of the individual single-objective optimization problems, 2) determining the best and worst solutions for each of the objective functions, 3) using these solutions as the boundaries of the fuzzy ranges in the corresponding fuzzy optimization problem, and 4) solving the resulting fuzzy optimization problem.

The membership function of a fuzzy objective function is constructed as

$$\mu_{f_i}(X) = \begin{cases} 0 & \text{if } f_i(X) \ge f_i^{\max} \\ \left[\dfrac{-f_i(X) + f_i^{\max}}{f_i^{\max} - f_i^{\min}}\right] & \text{if } f_i^{\min} < f_i(X) < f_i^{\max} \\ 1 & \text{if } f_i(X) \le f_i^{\min} \end{cases}$$

$$i = 1,2,\ldots,k \tag{13}$$

where $f_i^{\min} = \min_j f_i(X_j^*)$, $f_i^{\max} = \max_j f_i(X_j^*)$, and X_j^* is the optimum design vector of the jth objective function. Equation (13) models a linear (risk-neutral) membership function for the objective function. When the fuzzy constraints are stated as

$$g_j(X) \le b_j + d_j, \qquad j = 1,2,\ldots,m \tag{14}$$

where d_j denotes the distance by which the boundary of the jth constraint is moved, the linear membership function of the jth constraint is constructed as

$$\mu_{g_j}(X) = \begin{cases} 0 & \text{if } g_j(X) \geq b_j + d_j \\ 1 - \left[\dfrac{g_j(X) - b_j}{d_j}\right] & \text{if } b_j < g_j(X) < b_j + d_j \\ 1 & \text{if } g_j(X) \leq b_j \end{cases}$$

$$j = 1,2,\ldots,m \qquad (15)$$

Once the membership functions of the objectives and the constraints, i.e., μ_f and μ_g are known, the fuzzy optimization problem [Eq. (4)] can be posed as an equivalent crisp optimization problem as follows:

Find X and λ, which

$$\text{Maximize } \lambda$$

subject to

$$\lambda \leq \mu_{f_i}(X), \qquad i = 1,2,\ldots,k \qquad (16)$$

$$\lambda \leq \mu_{g_j^u}(X), \qquad j = 1,2,\ldots,m \qquad (17)$$

$$\lambda \leq \mu_{g_j^l}(X), \qquad j = 1,2,\ldots,m \qquad (18)$$

This problem can be solved using standard single-objective nonlinear programming techniques. A commercially available optimization library ADS[15] has been used for this purpose.

Numerical Results

The effectiveness of the fuzzy optimization techniques presented in the previous section is demonstrated through application to two types of helicopter design problems. The first problem deals with the determination of optimum flight parameters to accomplish a specified mission in the presence of three competing design objectives. The second problem addresses the optimum design of the main rotor of a helicopter to accomplish a specified mission in the presence of eight different objective functions. These two problems, with only single objective functions were originally considered by Bennet,[17] and the analysis programs reported therein have been modified for use in the present work.

Flight Trajectory Optimization

Flight profile optimization addresses the need for determining optimum flight parameters to accomplish a specified mission for a given payload. The mission planning task is to select, prior to the flight, the altitude-speed profile and the initial fuel load for the mission. The objectives, for example, may be to minimize fuel cost, flight time, or total cost or to maximize range. Earlier attempts at solving such problems have employed the principles of optimal control theory.[16] However, if a mission can be discretized into a finite number of segments, and if the flight conditions remain constant over each segment, the flight trajectory optimization problem can be formulated as a standard mathematical programming problem. This approach has been adopted in the present work.

For any specified mission, the flight profile optimization program (FPOP) generates an initial mission description consisting of ten segments of equal length. This initial mission approximation assumes that the fuel tanks are full at takeoff. The power required and the fuel flow necessary for each segment are computed based on atmospheric properties such as air density, temperature, and wind speed. At the end of each mission segment, parameters such as helicopter altitude, fuel consumed, flight time, gross weight, and distance traveled are computed by FPOP. The performance characteristics of the helicopter are derived from actual test data and are expressed as follows:

1) The power coefficient C_p is expressed as a continuous function of advance ratio (μ) for discrete values of thrust coefficient C_t using a polynomial representation.

2) The engine fuel flow is expressed as a function of shaft horsepower for discrete values of density altitude using a seventh-order polynomial representation.

3) A seventh-order polynomial is used to express hover power coefficient as a continuous function of thrust coefficient for hover OGE and hover IGE.

The flight parameters such as the initial fuel load, indicated airspeed at the beginning of each of the ten segments, and the rate of climb at the beginning of the first nine segments are varied during the design procedure. The rate of climb or descent for the fifth segment of a two-way mission, and the rate of descent for the tenth segment are not true independent variables. The values of these parameters are computed based on the altitude values at the beginning of the fifth (or tenth) segment and the destination altitude. From the initial profile determined by FPOP, and NLP algorithm iterates the flight parameters until an optimum profile satisfying the imposed constraints is determined.

The design variables for this problem are 1) the initial fuel load, 2) indicated airspeed at the beginning of each of the ten segments, and 3) the rate of climb (or descent) at the beginning of the first nine segments for a one-way mission; or the rate of climb (or descent) for segments 1-9 for a two-way mission. The behavior constraints considered in the problem formulation include the following:

1) Horsepower required for each segment ≤ horsepower available

2) Indicated airspeed ≤ V_{NE} for each segment

3) Altitude for each segment ≤ maximum altitude

4) Takeoff weight ≤ maximum takeoff gross weight: or takeoff weight ≤ maximum weight for hover OGE; or takeoff weight ≤ maximum weight for hover IGE

5) Fuel required ≤ fuel available

6) Error in terminal altitude is within the prescribed limits.

The side constraints (on design variables) include the following:

1) Rate of climb for each segment ≤ maximum rate of climb

2) Rate of descent for each segment ≤ maximum rate of descent.

Three objective functions, namely, the minimization of fuel cost, flight time, and total cost are considered, with prescribed values for payload and range. The design data for the problem are given in Table 1.

The result from single-objective optimizations yield a $k \times k$ matrix $[M]$ defined as follows:

$$[M] = \begin{bmatrix} f_1(X_1^*) & f_2(X_1^*) & \cdot & f_k(X_1^*) \\ f_1(X_2^*) & f_2(X_2^*) & \cdot & f_k(X_2^*) \\ \cdot & \cdot & \cdot & \cdot \\ f_1(X_k^*) & f_2(X_k^*) & \cdot & f_k(X_k^*) \end{bmatrix} = \begin{bmatrix} 383.37 & 2.7485 & 1372.83 \\ 427.39 & 2.3675 & 1279.70 \\ 423.13 & 2.3592 & 1272.43 \end{bmatrix} \qquad (19)$$

Once the best and the worst values of each of the three objectives are identified, the membership functions of the three objectives are constructed as follows:

$$\mu_{f_1}(X) = \begin{cases} 0 & \text{if } f_1(X) \geq 427.39 \\ \left[\dfrac{-f_1(X) + 427.39}{44.02}\right] & \text{if } 383.37 < f_1(X) < 427.39 \\ 1 & \text{if } f_1(X) \leq 383.37 \end{cases} \tag{20}$$

$$\mu_{f_2}(X) = \begin{cases} 0 & \text{if } f_2(X) \geq 2.7485 \\ \left[\dfrac{-f_2(X) + 2.7485}{0.3893}\right] & \text{if } 2.3592 < f_1(X) < 2.7485 \\ 1 & \text{if } f_2(X) \leq 2.3592 \end{cases} \tag{21}$$

$$\mu_{f_3}(X) = \begin{cases} 0 & \text{if } f_3(X) \geq 1372.83 \\ \left[\dfrac{-f_3(X) + 1372.83}{100.4}\right] & \text{if } 1272.43 < f_3(X) < 1372.83 \\ 1 & \text{if } f_3(X) \leq 1272.43 \end{cases} \tag{22}$$

The membership functions of the design variables are constructed using the bounds given in Table 2 as

$$\mu_{x_j^u}(X) = \begin{cases} 0 & \text{if } x_j \geq 174.9 \\ 1 - \left(\dfrac{x_j - 159.0}{15.9}\right) & \text{if } 159.0 < x_j < 174.9 \\ 1 & \text{if } x_j \leq 159.0 \end{cases}$$

$$j = 2,\ldots,11 \tag{23}$$

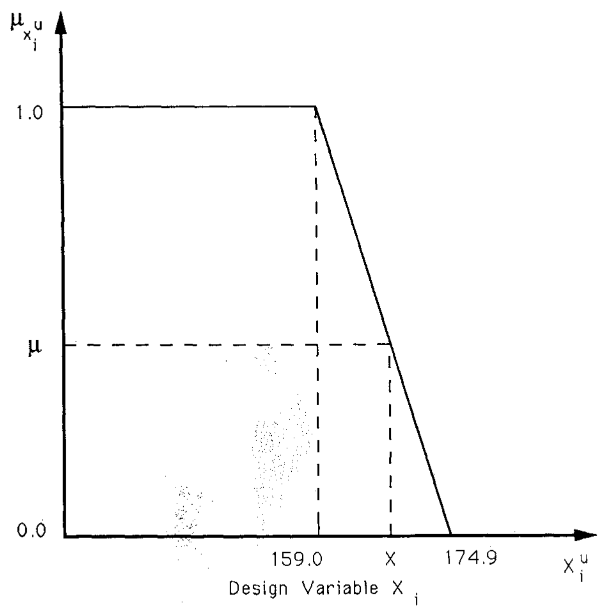

Fig. 1 Membership funciton for upper bound on the design variable x_i.

$$\mu_{x_j^l}(X) = \begin{cases} 1 & \text{if } x_j \geq 70 \\ \left(\dfrac{x_j - 63}{7}\right) & \text{if } 63 < x_j < 70 \qquad j = 2,\ldots,11 \\ 0 & \text{if } x_j \leq 63 \end{cases} \tag{24}$$

Membership functions of the remaining design variables are constructed in a similar fashion. Figures 1-3 depict the membership functions corresponding to the upper and lower bounds on design variables x_2-x_{11} and the objective function f_1. By permitting a 10% leeway, membership functions of the 34 behavior constraints present in the problem formulation are also constructed. It may be noted that (for this problem)

Table 1 Baseline mission profile parameters[17]

Mission Requirements	
Payload: 2200 lb	Range: 180 n.m.
Crew weight: 190 lb	Fuel reserve: 333 lb
Altitude at takeoff: SLS	Altitude of destination: SLS
Maximum flight altitude: 8000 ft	Weight empty: 11,500 lb
Maximum rate of climb: 1200 ft/min	Maximum descent = −600 ft/min
Fuel cost: $1.25/gal	Maintenance cost = $360/hr
Max internal gross weight: 17,500 lb	Fuel capacity: 2958 lb
Hover transmission limit: 2350 hp	Transmission limit: 1950 hp

Table 2 Bounds on design variables for flight profile optimization

Design variable	Crisp lower bound	Crisp upper bound	Fuzzy lower bound	Fuzzy upper bound
x_1	10.0	2957.7	9.0	3253.5
x_2-x_{11}	70.0	159.0	63.0	174.9
$x_{12}-x_{15}$	−600.0	1200.0	−660.0	1320.0
x_{16}	99.9	100.1	99.9	100.1
$x_{17}-x_{20}$	−600.0	1200.0	−660.0	1320.0

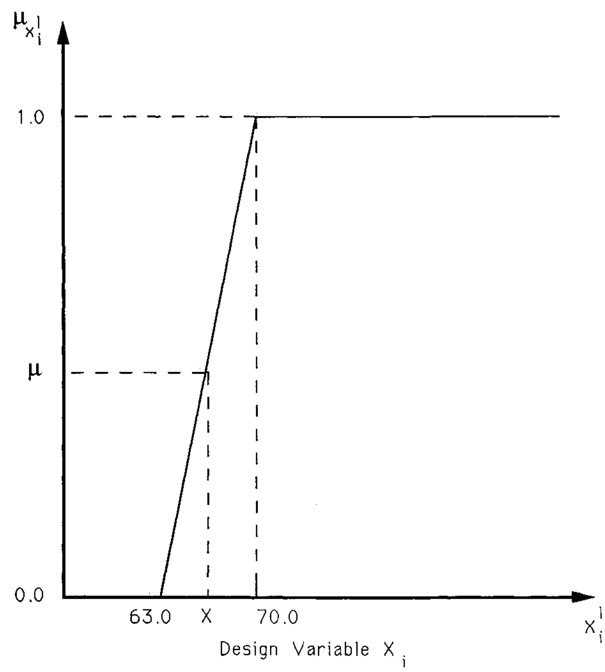

Fig. 2 Membership function for lower bound on the design variable x_i.

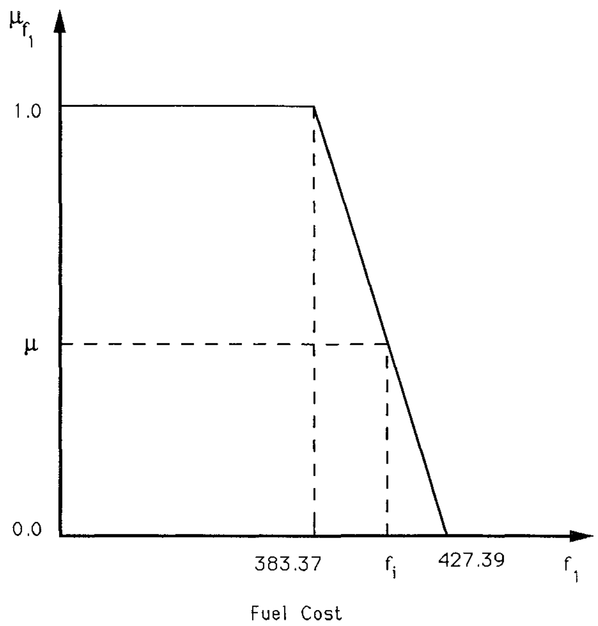

Fig. 3 Membership function for the first objective, f_1.

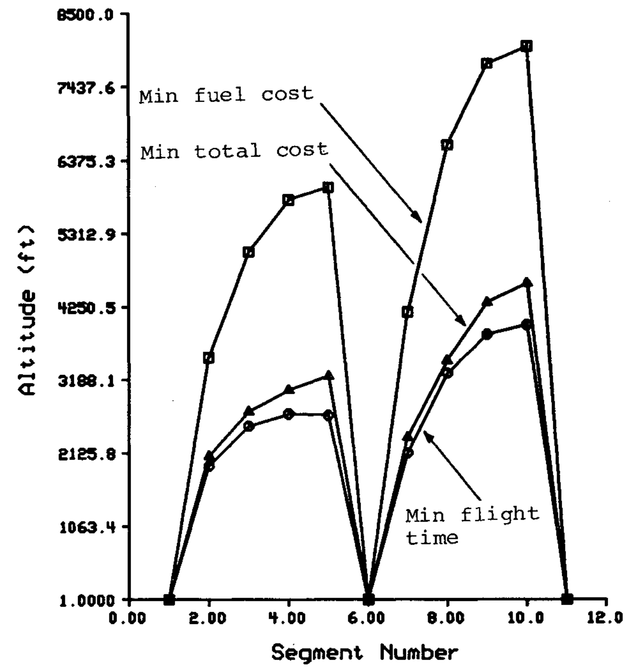

Fig. 4 Flight trajectories for single-objective optimizations.

design variable x_{16} is not truly an independent design variable. Its value is determined by the altitude at the beginning of the fifth segment and by the destination altitude.

Since a design vector with the highest degree of membership in the fuzzy decision set is required, the fuzzy mulitobjective optimization problem is formulated as

Find X and λ, which

$$\text{Maximize } \lambda$$

subject to

$$\lambda \leq \mu_{f_i}(X), \qquad i = 1,2,3 \tag{25}$$

$$\lambda \leq \mu_{g_j}(X), \qquad j = 1,2,...,34 \tag{26}$$

$$\lambda \leq \mu_{x_j^l}(X), \qquad j = 1,...,15,17,...,20 \tag{27}$$

$$\lambda \leq \mu_{x_j^u}(X), \qquad j = 1,...,15,17,...,20 \tag{28}$$

The preceding problem has a total of 21 design variables and 75 constraints. The numerical results obtained by solving this mathematical programming problem are presented in Table 3. The optimum solution yields an overall satisfaction level (λ) of 79.9%. The optimum flight trajectories obtained using single- and multiple-objective optimization techniques are presented in Figs. 4 and 5, respectively. It can be seen from Fig. 4 that when the objective is to minimize the fuel cost, the optimum flight parameters require that the helicopter be flown at altitudes ranging from 5000 to 8000 ft for a good part of the total flight path. However, when the objective is to minimize the flight time or the total cost, the helicopter is flown at altitudes below 4500 ft, both during the outbound and return segments of the mission. Also, the rates of climb and descent for the return segment of the mission are higher compared to the corresponding values during the outbound flight because some fuel has been consumed during the forward leg and the payload has been delivered.

When all three objectives are considered simultaneously, it is observed that crisp multiobjective optimization (goal attainment) schemes yield flight parameters requiring that the helicopter be operated under 5000 ft. When the flight trajectory optimization problem is solved using the techniques of fuzzy optimization, the helicopter altitude during each segment of the flight path exceeds the corresponding values given by crisp (single-objective and multiobjective) optimization schemes by as much as 1800–4000 ft (Fig. 5).

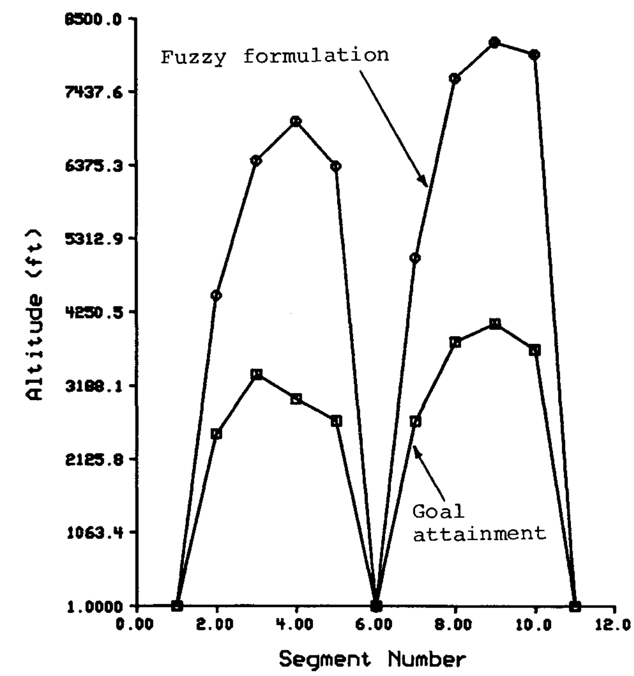

Fig. 5 Flight trajectories for goal attainment and fuzzy formulations.

Optimum Design of the Main Rotor

A viable and efficient design of the main rotor of a helicopter requires an integration of several analytical disciplines, such as aerodynamics, structures, noise, and mission analysis. The application of optimization techniques to rotor design problems can be broadly divided under three categories: 1) global performance design of rotor, 2) structural design of blades, and 3) aerodynamic and acoustic design.

The equations used for blade design in this work are limited to momentum theory considerations to minimize the required input information, procedural detail, and computational complexity without markedly compromising the utility of the solutions. The analysis scheme incorporates mathematical models for hover, vertical flight, and forward flight conditions. Mathematical models for engine performance, fuel consumption, and aircraft group weights are also included in the analysis procedure. The design procedure utilizes the following computation scheme: Initially, the engine is sized to meet the most demanding power requirement among the hover, vertical climb, and high-speed segments of the mission, including the operation of a multiengined helicopter with one engine inoperative. The fuel weight and the aircraft component group weights are next computed, thus enabling the determination of available payload capacity. A comparison of the available payload with the required payload leads to a new gross weight estimation. The analysis loop is re-entered with this new gross weight estimate, and the iteration is continued until a gross weight compatible with the required payload is determined.

The power required is viewed as compromising three power-absorbing components:

$$SHP = HP_{MR} + HP_{TR} + HP_{XM} \tag{29}$$

where SHP is the required shaft horsepower, and HP_{MR}, HP_{TR}, and HP_{XM} are the power required by the main rotor, tail rotor, and transmission system, respectively. The power required by the main and tail rotors is further subdivided into several components. For example, the power required by the main rotor for level forward flight is expressed as the sum of parasite, profile, induced power components, and additional power required as a result of compressibility and stall effects. The tail rotor power is taken to be the sum of tail rotor profile and induced power requirements. Power requirements for other flight conditions are computed in a similar fashion. The power lost in the transmission system is expressed as a constant percentage of the power required by the main and tail rotors.

The weight is computed based on a statistical analysis of the weights of 59 different helicopters. The weight breakdown includes the weight of the main rotor, body, propulsion and transmission systems, instrumentation, landing gear, and tail rotor. The equations used for the component weights are derived from a multiple regression analysis of existing helicopters. Using the weight breakdown and cost per pound of various components, the cost of the proposed helicopter is determined. The flyover noise level is expressed as a function of tip speed, Mach number, and gross weight number. The handling qualities of the helicopter are determined by the lock number and the autorotation parameter. The fuel required for a mission is computed in two different manners. The easiest way is to specify the required payload and range for the mission. The second method of specifying mission is to define the mission at ten individual segments. At each segment, it is necessary to specify a time at that segment, atmospheric properties, velocity, and climb conditions. Upon integrating the fuel flow rate over the duration of the segment, the fuel burned during any mission segment is computed. The second method permits a much more detailed description of the helicopter mission. The cruise speed, dash speed, endurance, as well as hover ceiling, are also computed as part of the analysis procedure.

The single-rotor helicopter design and performance estimation program (SSP1) developed by Schwartzberg[18] has been used in this work to design the main rotor blades. The main rotor radius, chord, twist, and tip speed are treated as design variables for this problem. The following inequality constraints are considered in the problem formulation:

1) Fuel required ≤ fuel available.
2) Required payload ≤ available payload.
3) Hover blade loading coefficient ≤ specified value.
4) Maximum advancing tip Mach number ≤ specified value.
5) Blade loading in forward flight ≤ specified value.
6) Hover horsepower ≤ specified value.
7) Hover horsepower for OEI ≤ specified value.
8) Horsepower for forward flight OEI ≤ specified value.
9) Horsepower for maximum speed ≤ specified value.
10) Horsepower for maximum sustained G level ≤ specified value.
11) Autorotation index ≤ specified value.
12) Maximum flyover noise level ≤ specified value.

The objective functions considered include the following:

1) Minimum gross weight.
2) Minimum manufacturing cost.
3) Minimum empty weight.
4) Minimum mission fuel.
5) Maximum endurance limit.
6) Maximum dash speed.
7) Maximum hover ceiling.
8) Minimum noise level.

The design data for the problem are given in Table 4.

Table 3 Mission parameters for fuzzy optimization

Design variables:

Airspeeds, knots: 140.3, 107.4, 95.3, 99.0, 148.0, 109.8, 93.5, 90.8, 92.6

Rates of climb, ft/s: 297.8, 129.1, 38.2, −43.5, −434.8, 352.3, 182.0, 36.7, −12.4, −564.1

Initial fuel = 2418.4 lb $\lambda^* = 0.7987$

Objective functions:

Fuel cost = \$392.29, flight time = 2.438 h, total cost = \$1269.81

Table 4 Design parameters for a typical small helicopter[17]

Baseline airframe: OH-58A
Baseline engine: T63
Required range: 300 n.m. at SLS
Required payload: 970 lb

Design variables and bounds:

	Minimum value	Initial value	Maximum value
Tipspeed, ft/s	625.0	642.14	800.0
Radius, ft	10.0	14.37	18.0
Chord, ft	0.5	0.59	2.0
Twist, deg	−20.0	−16.84	0.0

Engine sizing points:

1. Hover OGE at 6000 ft/37.6°F; hp avail = 350
2. Level flight speed = 132 knots at SLS; hp avail = 302

Design constraints:

1) Engine power required for engine sizing point 1
2) Engine power required for engine sizing point 2
3) Max advancing tip Mach number < 0.95
4) Hover blade loading coefficient 2 C_T < 0.18
5) Forward flight blade loading coefficient < 0.50–0.46μ
6) Minimum t/K > 0.5
7)Maximum noise level < 93 dB

Using the results from single-objective optimizations, the matrix $[M]$, defined in Eq. (19), is constructed as follows:

$$[M] = \begin{bmatrix} 2943 & 340703 & 1076 & 446 & 3.39 & 162.2 & 6040.7 & 89.1 \\ 2493 & 340697 & 1076 & 447 & 3.39 & 162.2 & 6039.7 & 89.1 \\ 2493 & 340747 & 1076 & 447 & 3.39 & 162.2 & 6042.1 & 89.1 \\ 2572 & 371346 & 1167 & 435 & 3.43 & 164.9 & 7169.1 & 89.2 \\ 2764 & 431364 & 1345 & 450 & 3.55 & 158.5 & 8815.4 & 91.3 \\ 2569 & 368691 & 1162 & 436 & 3.38 & 165.7 & 6144.7 & 88.4 \\ 2742 & 427478 & 1333 & 439 & 3.55 & 161.7 & 8933.1 & 90.7 \\ 2588 & 374821 & 1181 & 437 & 3.38 & 165.5 & 6047.4 & 88.4 \end{bmatrix} \tag{30}$$

Once the best and the worst values for each of the eight objectives are identified, the membership functions of the eight objectives are constructed as follows:

$$\mu_{f_1}(X) = \begin{cases} 0 & \text{if } f_i(X) \geq 2764 \\ \left[\dfrac{-f_1(X) + 2764}{271}\right] & \text{if } 2493 < f_1(X) < 2764 \\ 1 & \text{if } f_i(X) \leq 2493 \end{cases} \tag{31}$$

$$\mu_{f_2}(X) = \begin{cases} 0 & \text{if } f_2(X) \geq 431364 \\ \left[\dfrac{-f_2(X) + 431364}{90667}\right] & \text{if } 340697 < f_2(X) < 431364 \\ 1 & \text{if } f_2(X) \leq 340697 \end{cases} \tag{32}$$

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$$\mu_{f_8}(X) = \begin{cases} 0 & \text{if } f_8(X) \geq 91.3 \\ \left[\dfrac{-f_8(X) + 91.3}{2.9}\right] & \text{if } 88.4 < f_8(X) < 91.3 \\ 1 & \text{if } f_8(X) \leq 88.4 \end{cases} \tag{33}$$

Using the bounds on design variables indicated in Table 5, the membership function of design variable x_1 is constructed as follows:

$$\mu_{x_1^u}(X) = \begin{cases} 0 & \text{if } x_1 \geq 880 \\ 1 - \left(\dfrac{x_1 - 800}{80}\right) & \text{if } 800 < x_1 < 880 \\ 1 & \text{if } x_1 \leq 800 \end{cases} \tag{34}$$

Table 5 Bounds on the design variables for main rotor design

Design variable	Crisp lower bound	Crisp upper bound	Fuzzy lower bound	Fuzzy upper bound
x_1	625.0	800.0	562.5	880.0
x_2	10.0	16.0	9.0	17.6
x_3	0.5	2.0	0.45	2.2
x_4	−20.0	0.0	−22.0	0.0

Table 6 Main rotor parameters for fuzzy optimization

Design variables:	
MR radius = 13.94 ft	MR chord = 0.52 ft
MR twist = −19.98 deg	Tip speed = 656.12 ft/s
λ^* = 0.361	
Objective functions:	
Gross weight = 2600 lb	Total cost = $381,654
Empty weight = 1197 lb	Fuel weight = 433 lb
Endurance = 3.46 h	Dash speed = 165.0 knots
Hover ceiling = 7720 ft	Noise level = 89.4 dB

$$\mu_{x_1^l}(X) = \begin{cases} 1 & \text{if } x_1 \geq 625.0 \\ \left(\dfrac{x_1 - 562.5}{62.5}\right) & \text{if } 562.5 < x_1 < 625.0 \\ 0 & \text{if } x_1 \leq 562.5 \end{cases} \tag{35}$$

Membership functions of the remaining design variables ($x_2 - x_4$) are constructed in a similar fashion. By permitting a 10% leeway, membership functions corresponding to the 12 behavior constraints present in the formulation are also constructed. Once the membership functions of all the fuzzy objectives and constraints are determined, the resulting fuzzy optimization can be stated as

Find X and λ which

$$\text{Maximize } \lambda$$

subject to

$$\lambda \leq \mu_{f_i}(X), \qquad i = 1,2,\ldots,8 \tag{36}$$

$$\lambda \leq \mu_{g_j}(X), \qquad j = 1,2,\ldots,12 \tag{37}$$

$$\lambda \leq \mu_{x_j^l}(X), \qquad j = 1,\ldots,4 \tag{38}$$

$$\lambda \leq \mu_{x_j^u}(X), \qquad j = 1,\ldots,4 \tag{39}$$

The preceding problem has a total of 5 design variables and 28 inequality constraints. The results obtained by solving this mathematical programming problem are presented in Table 6. The optimum solution corresponds to an overall satisfaction level of 36.1%. It is observed from the results obtained by solving crisp single and multiobjective optimization problems that the design variable x_3 (linear twist of the main rotor blades) is always at its lower bound (-20 deg) at the optimum solution. This would seem to indicate that when the lower bound is relaxed in the fuzzy formulation, the linear twist of the main rotor blades may go down even farther. This is contrary to what is obtained when the fuzzy optimization problem is solved. The linear twist for the fuzzy optimum design is still close to -20 deg.

Conclusions

For the first design example dealing with the determination of optimum flight parameters to accomplish a specified mission, it may be noted from Eq. (19) that the three objectives considered are conflicting. A design with minimum fuel cost design yields poor values for flight time and total mission cost. Attempting to achieve a design with improved values for flight time and/or total cost results in a high rate of fuel consumption. Multiobjective optimization techniques presented in this work are able to achieve a compromise by permitting a tradeoff between these conflicting pairs of objectives.

It was recognized that considerable fuzziness exists in the problem definition and in the mission requirements and constraints. To model this imprecise information in a quantitative manner, the tools of fuzzy set theory are used. The fuzzy formulation yields the best of values for the three objectives at the optimum solution. In fact, the optimum value of f_3 (total cost) for fuzzy multiobjective formulation with relaxed constraints is *lower* than the value obtained when f_3 is considered alone. All the improvements are possible at the expense of relaxation of the maximum altitude constraint. The maximum altitude attained by the helicopter using fuzzy formulation is 8160 ft compared to a maximum value of 8000 ft for the crisp case. There is no need to change any of the other helicopter parameters, such as horsepower required and fuel tank capacity.

For the second design problem, it may be noted from Eq. (30) that when the main rotor is designed for minimum gross weight, the resulting design also has the minimum total cost and minimum empty weight. This design, however, has a low hover ceiling and a low endurance limit. Attempting to maximize the endurance limit and/or the hover ceiling results in a noisy design with high gross weight and a high cost of manufacturing. Because of the conflicting nature of these multiple objectives, single-objective optimization techniques are unable to obtain a satisfactory solution. Multiobjective fuzzy optimization techniques presented in this work are able to achieve a superior compromise between these conflicting pairs of objectives (Table 6).

To summarize, the fuzzy optimization techniques presented in this work are expected to be extremely useful during the initial stages of conceptual design of engineering systems where the design goals and constraints have not been clearly identified or stated. These techniques can effectively model the vague and imprecise information present in the objective function and constraints to formulate fuzzy goals and constraints. These models can be used efficiently and effectively for decision-making problems in ill-structured situations. Further, since these techniques result in a unified approach to a decision-making process in the sense that there is no longer any distinction between the goals and the constraints, they are, in general, able to achieve a superior solution compared to other multiobjective optimization techniques.

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